S O L U T I O N S

The problems in this book are designed to make you THINK about the ideas being presented. Understand the ideas and you should be able to do the problems.

A couple of suggestions:

1.) Study the book. Try the examples. Once done, attempt the chapter-end problems.

2.) You can use the book, friend, whatever, in trying the QUESTIONS, but do the chapter-end PROBLEMS *without* using aids. That is, treat them like *test* problems. Above all, do not simply look at a PROBLEM, wait fifteen seconds, decide you can't do it, then turn to the *Solutions*.

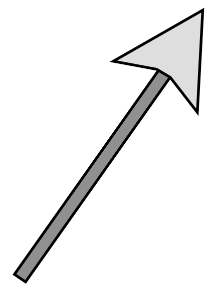
3.) If you are confused, re-study the book, come see me, or talk to a knowledgeable friend. Use the *Solutions* for chapter-end PROBLEMS only after all else has failed. Otherwise, all you will be doing is riding on *my* ability to dispatch the pesky critters. Believe me, that is not going to help you in the long run.

CHAPTER 3 -- MATH REVIEW

QUESTION SOLUTIONS

3.1) Is three plus four always equal to seven? Explain.

Solution: If the numbers are scalars written in base 10, the answer is *yes* (if the numbers are in any other base above base six, the answer is NO, though that isn't something most students would notice, and it really has nothing to do with physics or physics related math). More to the point, if the numbers are associated with VECTORS, the answer is NO. That is, adding two vectors that are in opposite directions, or are at right angles to one another, or are in any orientation other than being aligned, is going to produce a summation that does not add like a scalar.

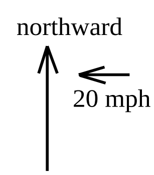


3.2) You're bored. You find yourself casually perusing the math book your parents keep for guests on their living room table (. . . hey, it could happen . . .). You come across a spectacularly intriguing vector in the book. You're not doing much, so you decide to call your friend, Hilda, to let her know about this amazing vector you've just met. She gets excited and, being the pushy sort, wants to know more about the vector. Where's it from? How big is it? What's it look like? There are two fairly standard ways you could describe your vector so that Hilda could recreate it for herself. *For each approach*, what information would you have to provide for her to do so?

Solution: One way is to use a POLAR NOTATION and give the magnitude of the vector and the angle the vector makes with some reference line (the *+x axis* is usually used as the norm). The other possibility is to set up a coordinate axis at the tail of the vector and use a UNIT VECTOR NOTATION by specifying the x and y components (i.e., how far along the *x axis* and the *y axis* one must move to proceed from the tail to the tip of the vector).

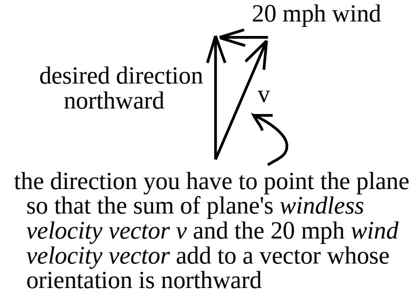
3.3) Why is it easy to add vectors in *unit vector notation* and not generally easy to add vectors in *polar notation*?

Solution: Unit vector notation breaks vectors into components oriented along specific directions. Adding two vectors in such a notation simply requires that all of the *x components* be added together and all of the *y components* be added together. Polar notation doesn't work like that. It identifies the vector's direction using an angle. Trying to scalar add two vectors at different angles is almost impossible unless you use a graphical method or convert them to unit vector notation.



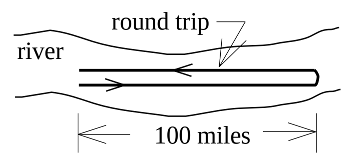
3.4) You are flying an airplane. There is a 20 mph breeze blowing to the west. What information do you need to determine the direction you must nose your plane if its resultant motion is to be directly northward? With that information, how would you solve the problem?

Solution: You need to know how fast the plane moves relative to the ground in



a windless situation (call this v). The problem states that you want to go north but are being pushed west (see original sketch). To counteract this, you have to nose the plane east of north. By how much? I'd do this graphically. Start with your *windless velocity* vector (its magnitude is v--you don't yet know its direction as that's the question). In what direction would you have to orient that vector, drawn to scale, so that when you graphically added it to a scaled westerly vector of magnitude 20 mph you get a resultant vector that is oriented northward. Using a protractor to determine that direction solves your problem.

3.5) You're in a motorboat. On flat (stationary) water, the boat's maximum speed is v_o. The river you are traveling on flows with speed $v_o/2$. You have to travel 100 miles up the river and 100 miles back down to your start position. If you made the round trip on flat water, it would take t hours to do the round trip. On this river, the round trip will take: (a) the same, (b) more, or (c) less time? Explain.



Solution: The answer is *b*. It is tempting to think that the decrease in speed while moving up river against the current is counteracted by the increase of speed while moving down river with the current. In fact, those two speeds *do* average to the flat water speed. The problem is that the amount of *time* the boat travels down river at higher speed is less than the amount of time the boat has to travel up stream at slower speed, so the overall elapsed time is greater in the river with current.

3.6) You are given the *dot product* and *cross product* for the same two vectors. What clever thing could you do to determine the angle between the two vectors?

Solution: You could divide one into the other. The *dot product* is defined as the product of the magnitude of the two vectors times the *cosine of the angle between the two vectors*. The magnitude of the *cross product* is defined as the product of the magnitude of the vectors times the *sine of the angle between the two vectors*. If you divide the *cross product* by the *dot product*, you will end up with a number that is equal to the sine divided by the cosine of the angle between the two vectors. Through trig identities, that is equal to the tangent of the angle . . . the inverse tangent of which is easily determined with a calculator.

3.7) You are given the *cross product* between a known vector ***A*** and a vector ***B*** whose magnitude you know but whose direction you don't know. From this, what can you tell about the direction of the unknown vector? Explain.

Solution: Doing a *cross product* between two vectors produces a third vector whose direction is ALWAYS perpendicular to the plane defined by the original two vectors. That means that if you are given a *cross product*, direction included, you know the plane in which the original vectors must have resided. You know the magnitudes A, B, and that of ***AxB*** . You also know that the magnitude of ***AxB*** is equal to the product of the magnitudes of the two vectors (AB) times the sine of the angle between the two. With that, you can determine the angle between the two vectors. Knowing the angle between ***A*** and ***B***, you can almost determine the orientation of ***B***. Almost, because ***B*** could be oriented clockwise or counterclockwise from ***A***. To determine which, note that the *right hand rule* determines the direction of a *cross product*. Using the right hand rule in conjunction with your knowledge of the *direction* that that rule should give you (i.e., the direction of the known *cross product*) will allow you to determine the orientation of ***B*** relative to ***A*** (i.e., if you use your hand to cross ***A*** into ***B***, assuming that ***B*** is counterclockwise from ***A***, you will end up with a *cross product* direction that is in the ***+k*** direction--if the *cross product* vector you were given in the problem is negative, you know that ***B*** must not be oriented that way).

3.8) You are told that the *cross product* between two vectors is zero. What do you know about the two vectors?

Solution: For a *cross product* to be zero, the involved vectors must be colinear. That is, the angle between the two vectors must be 0^o or 180^o so that the *sine of the angle* is zero.

3.9) What does the direction of a *cross product* tell you?

Solution: The direction of a *cross product* is ALWAYS perpendicular to the plane defined by the two vectors in question, but the significance of that direction depends upon the vectors and the situation being modeled. If, for instance, you cross a force vector into a displacement vector to determine a torque, the direction of the *cross product* gives you the direction of the *axis about which the torque is being applied* (in most cases, this is the axis of rotation). If, as a second example, you cross a velocity vector of a positively charged particle that is moving through a magnetic field into the magnetic field vector, the direction of the *cross product* gives you the direction of the magnetic force on the particle.

3.10) What is *always* true about the direction of a *cross product*?

Solution: The direction of a *cross product* is ALWAYS perpendicular to the plane defined by the vectors involved.

3.11) What does a *cross product* really do for you?

Solution: A *cross product* yields the product of the magnitudes of one vector and the perpendicular component of the second vector (*the perpendicular component* of the second vector is the component of the second vector that is perpendicular to the line of direction of the first vector).

3.12) You are told the *dot product* between two vectors is zero. What do you know about the two vectors?

Solution: For a *dot product* to be zero, the vectors involved must be perpendicular to one another. In that way, the *cosine of the angle* will be zero.

3.13) What does a *dot product* really do for you?

Solution: A *dot product* yields the product of the magnitudes of one vector and the parallel component of the second vector (*the parallel component* of the second vector is the component of the second vector that is parallel to the line of the first vector).

3.14) Why would you laugh in the face (ha ha) of someone who asked what the direction of a *dot product* tells you?

Solution: *Dot product*s are scalars. They don't have directions associated with them (they do have signs, but the signs have nothing to do with direction).

3.15) Can a *dot product* be negative? If so, what would a negative *dot product* mean?

Solution: By simply executing its definition, the *dot product* between two vectors that are separated by more than 90^o is negative (cosine of an angle between 90^o and 180^o is negative), so the answer to the first part is *yes*. What negative signs actually mean depends upon what the vectors are and what situation is being modeled (it has nothing to do with *direction* as *dot products* are not vectors). For instance, the amount of energy put into or taken out of a system--a body moving over a tabletop, for instance--by a force $\boldsymbol{F}$ is defined as $\boldsymbol{F}\cdot\boldsymbol{d}$, where $\boldsymbol{d}$ is the distance over which the force is applied. If $\boldsymbol{F}$ is oriented opposite the direction of motion (hence, opposite the direction of $\boldsymbol{d}$), $\boldsymbol{F}$ opposes the motion and works to slow the body down (that is the same as saying that energy is being pulled out of the system). The angle between the two vectors is greater than 90^o, so the *dot product* is negative. The significance of the negative sign here? In this case it signifies the fact that energy is being pulled *out* of the system. If the force had been generally along the line of motion, energy would have been put *into* the system and the *dot product* would have been positive.

3.16) You can dot a vector into the results of a *cross product*, but you can't cross a vector into the results of a *dot product*. Why not?

Solution: A *cross product* yields a vector. You can, therefore, dot a vector into a *cross product* (vector) and end up with something that is kosher (i.e., another vector). A *dot product* produces a scalar. You can NOT, therefore, cross a vector into the results of a *dot product* as you can't cross a vector into a scalar.

3.17) You decide to convert vector $\boldsymbol{A}$ from unit vector notation to polar notation. You use your calculator to do the deed and you get a magnitude and an angle. Are you sure the magnitude and angle you get from your calculator are appropriate for the vector you are trying to characterize?

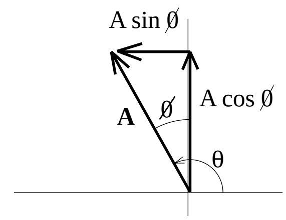
Solution: There is no problem with the magnitude--it will be correct no matter what. The angle is another story. It is determined by taking the inverse tangent of the ratio of A_y/A_x. The problem is that all you have given your calculator is a *ratio*. If that ratio is, say, positive, your calculator can't tell whether both components are positive or both components are negative--each will give a positive ratio. By the same token, if that ratio is negative, your calculator can't tell if A_y is the negative part or A_x is the negative part. As there is no getting around this identification problem, at least as far as your calculator is concerned, your calculator will always yield a solution that is either a positive first quadrant angle or a negative fourth quadrant angle. If, in fact, the vector you are interested in happens to be in the second quadrant, you have to manipulate your calculator-determined angle to get the correct angle (in that case, you would have to add 180^o to the negative calculator angle).

3.18) What's wrong with the notation $\boldsymbol{A} = -3\angle 25^o$?

Solution: In all probability, the individual who wrote $\boldsymbol{A} = -3\angle 25^o$ was trying to identify the negative of a vector ($\boldsymbol{B} = 3\angle 25^o$) by putting a negative sign in front of that vector and rewriting it as $-\boldsymbol{B} = \boldsymbol{A} = -3\angle 25^o$. This is incorrect. Flipping the vector over (that's what taking the negative of a vector does) means the angle has changed by 180^o. This vector should be written as $-\boldsymbol{B} = \boldsymbol{A} = 3\angle(180^o + 25^o) = 3\angle 205^o$.

3.19) In converting from polar notation to unit vector notation, the expression $\boldsymbol{A} = A\cos\theta\,\boldsymbol{i} + A\sin\theta\,\boldsymbol{j}$, where θ is the angle between the *+x axis* and the vector, works just fine. What's wrong with using it?

Solution: The expression works, but it's just one more thing for kids to memorize, then forget two weeks later. It's much better for students to get comfortable enough with vectors to be able to simply draw a little sketch of the vector in question, create a right triangle within the context of the coordinate axis used, then use sine or cosine trig functions to generate their own expression for each component (see example). This will initially be more work--if nothing else, one would have to look to see if a component should be positive or negative, then manually put in the appropriate signs--but in the long run it will give the student the ability to derive from scratch the appropriate expression for the components in a given situation (versus mindlessly sticking numbers into a memorized expression they may or may not remember down the line).



$\mathbf{A} = -(A \sin Ø)\mathbf{i} + (A \cos Ø)\mathbf{j}$

3.20) Assume you have a vector characterized in unit vector notation. You want to create a second vector that is equal to *minus* that first vector. How would you do that? If, instead, the original vector had been characterized in polar notation, how would you do that?

Solution: In Unit Vector Notation, simply multiplying each component by *-1* will yield the negation of the vector. What you are technically doing is multiplying each unit vector (i.e., $\boldsymbol{i}$, $\boldsymbol{j}$, or $\boldsymbol{k}$) by *-1*, effectively flipping them over so that they are oriented in the opposite direction. The reality of the situation, though, is that the signs get pulled out in front of each component so the vector looks like, say, $\boldsymbol{A} = -4\boldsymbol{i} + 6\boldsymbol{j}$ instead of $\boldsymbol{A} = 4(-\boldsymbol{i}) + 6(+\boldsymbol{j})$. The bottom line is that $-\boldsymbol{A}$ will be written as $4\boldsymbol{i} - 6\boldsymbol{j}$, which is to say it will look as though the *numbers* have been multiplied by *-1*. In Polar Notation, on the other hand, flipping a vector over (i.e., multiplying by *-1*) is done simply by adding 180^{o} to the angle.

PROBLEM SOLUTIONS

3.21) Both *Parts a* and *b* are straight vector addition, graphical style. You need a centimeter stick and protractor to do them.

a.) In this case, the velocity component contributed by the rower is a vector directed straight across the river. It has a magnitude of 5 miles per hour. The velocity component contributed by the river's motion is directed downward. It has a magnitude of 2 miles per hour. Vectorial addition of the two vectors yields the net velocity of the boat during the crossing (see Figure 1). A protractor should be used to measure the angle. It turns out to be -22^o (minus because it is *below* the reference line straight across the river). A centimeter stick measures the resultant at 5.4 centimeters. Multiplying by the scaling factor, we get a net velocity vector of 5.4 miles per hour.

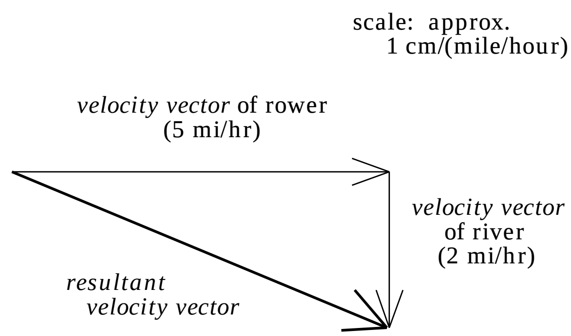


FIGURE 1

Note: The Xerox machine ever-so-slightly miniaturizes whatever it copies. When I used the original to make my measurements, I got the answer quoted above. If you use a centimeter stick and protractor on the Xeroxed copy, you will get an answer that is a bit off. If you do the problem on your own, you should get the same solution as I did to a good approximation.

b.) The direction of any object's *net velocity* is the same as the direction of the object's *net motion*. We want the net motion to be along the direction shown in Figure I on page 22. The question that is really being asked is: "What boat-velocity vector must exist if, when

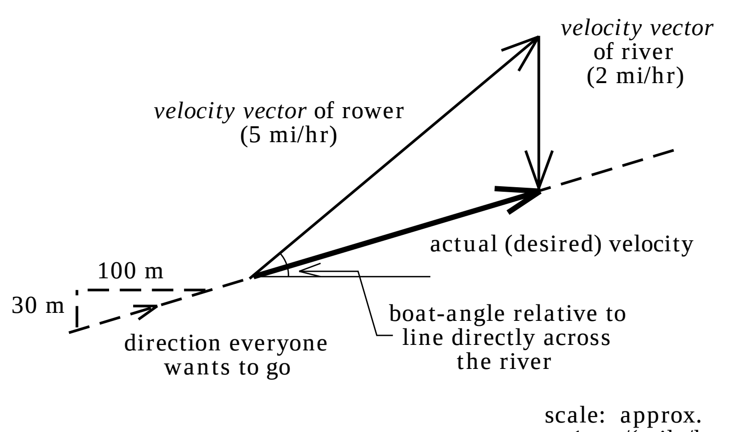


FIGURE 2

added to a water-velocity vector, yields the net-desired-boat-motion?" Clearly it must be upstream. Playing with a compass and protractor, the solution is approximately 40^o (see Figure 2 on the previous page) and the velocity is approximately 4 miles/hour.

3.22) This is also a problem for graphical vector addition.

a.) The vectors involved are shown to the right. A protractor was used to determine an angle of approximately 17^o north of west.

b.) The diagram is to scale. The resultant, measured by a centimeter stick, is approximately 5.6 centimeters. Multiplying by the scaling factor yields (5.6)(20) = 112 miles. This is the net displacement.

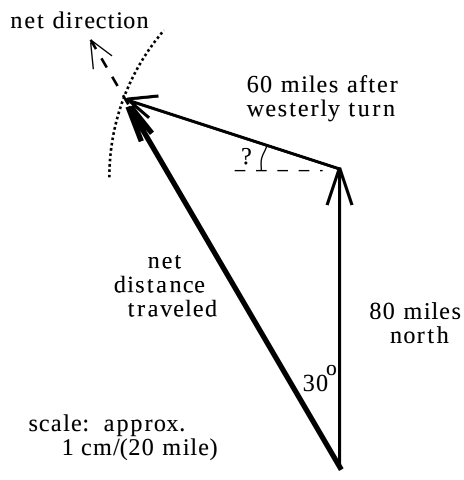


FIGURE 3

Note: Once again, if you use a centimeter stick and the sketch in Figure 3 to follow my steps in doing this problem, you will probably find your result and my result differing just a bit. I did the problem off the original drawing. Xeroxing slightly shrinks the material being copied.

3.23) The graph of vectors ***P*** and ***T*** are shown in Figure 4.

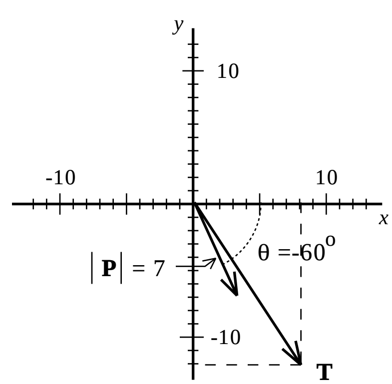


FIGURE 4

3.24) Using the graph given in problem-Figure II:

in unit vector notation: $\mathbf{A} = 2.25\mathbf{i} + .5\mathbf{j}$.
$\mathbf{C} = -.72\mathbf{i} - \mathbf{j}$.

in polar notation: $\mathbf{B} = 1.3\angle\ 125^o$.
$\mathbf{D} = 2.6\angle -20^o$.

Note: I've made the problem tricky as far as the two polar angles go. <u>All angles</u> must be measured either clockwise or counterclockwise FROM THE +*x* *AXIS*!

3.25) Each axis has its own scale. If the scales had been the same, relative lengths would have had significance. They aren't, hence they don't.

3.26) This is all straight vector manipulation. You should be able to do this kind of thing in your sleep!

a.) -(1/3)**A** = (-1/3)((-8**i** + 12**j**)
= (8/3)**i** - 4**j**.

b.) -6**E** = (-6)((12$\angle$225^o)
= (72$\angle$45^o).

Note: in polar notation, a *scalar* multiplies the *magnitude* of the vector while a *negative sign* adds (or subtracts) 180^o to the *angle* of the vector.

c.) **A** + **B** - **C** = (-8**i** +12**j**) + (-4**i** - 3**j**) - (5**i** + 6**j** -7**k**)
= -17**i** + 3**j** + 7**k**.

d.) A bit of trig and Figure 5 show ***E's*** unit vector equivalent. Written in *unit vector notation*:

E = -8.48**i** -8.48**j**.

Note: The negative signs had to be put in manually.

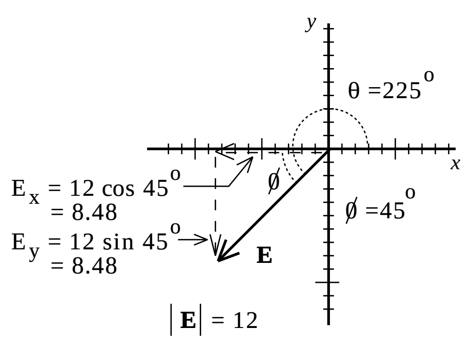


FIGURE 5

e.) A bit of trig and Figure 6 shows **F**'s *unit vector* equivalent to be:

F = -.518**i** + 1.93**j**.

Note: Again, the negative signs had to be placed manually.

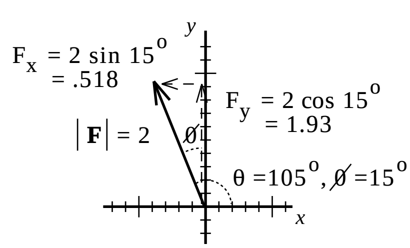


FIGURE 6

f.) The Pythagorean relationship, a bit of trig, and Figure 7 shows **A**'s *polar* equivalent to be:

$$\mathbf{A} = 14.4\angle 123.7^o.$$

Note: *Tan^{-1} (8/12) = 33.7^o*. Adding that to 90^o gives us the calculated 123.7^o. An alternate way to do the problem: take the actual components, minus signs and all, and determine *tan^{-1} (A_y/A_x)*. That is, *tan^{-1} (12/-8) = -56.3^o*. This is a *fourth quadrant angle,* whereas our vector is a *second quadrant angle*. To adjust the situation, we must add 180^o to get the appropriate 123.7^o angle. Either way will do.

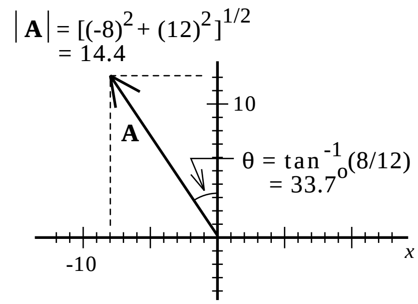


FIGURE 7

g.) The Pythagorean relationship, a bit of trig, and Figure 8 shows **B**'s *polar* equivalent to be:

$$\mathbf{B} = 5\angle 216.9^o.$$

Note: *Tan^{-1} (3/4) = 36.9^o*. Added to 180^o gives us the appropriate 216.9^o. An alternate way to do the problem: take the actual components, *minus signs and all*, and determine the *tan^{-1} (A_y/A_x)*, or *tan^{-1}(-3/-4) = 36.9^o*. This is a *first quadrant angle*, whereas our vector is a *third quadrant angle*. To adjust the situation, we must add 180^o to get the appropriate 216.9^o angle. Either way will do.

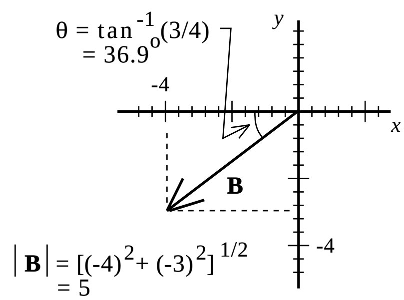


FIGURE 8

h.) $\mathbf{A}\cdot\mathbf{C} = A_xC_x + A_yC_y + A_zC_z$
$= (-8)(5) + (12)(6) + (0)(-7)$
$= 32.$

i.) $\mathbf{D}\cdot\mathbf{E} = |\mathbf{D}||\mathbf{E}|\cos\phi$
$= (7)\ (12)\cos 75^o$
$= 21.74.$

j.)

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 12 & 0 \\ -4 & -3 & 0 \end{vmatrix}$$

$= 0\mathbf{i} + 0\mathbf{j} + [(-8)(-3) - (12)(-4)]\mathbf{k}$
$= 72\mathbf{k}.$

k.)

$$\mathbf{C} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & -7 \\ -4 & -3 & 0 \end{vmatrix}$$

$= [(6)(0) - (-7)(-3)]\mathbf{i} + [(-7)(-4) - (5)(0)]\mathbf{j} + [(5)(-3) - (6)(-4)]\mathbf{k}$
$= -21\mathbf{i} + 28\mathbf{j} + 9\mathbf{k}.$

l.) $\mathbf{D}\text{x}\mathbf{E} = |\mathbf{D}||\mathbf{E}|\sin\phi\ (-\mathbf{k})$
$= (7)\ (12)\ \sin 75^o$
$= -81.1\mathbf{k}.$

Note: The *-**k** direction* came from using the *right-hand rule* on **D**x**E**.

3.27) A *dot product* gives you a scalar equal to the *magnitude of **A*** <u>times</u> the magnitude of the component of **B** *parallel to **A***. OR:

It gives you a scalar equal to the *magnitude of **B*** <u>times</u> the magnitude of the component of **A** *parallel to **B***.

3.28) A *cross product* gives you a vector whose magnitude is equal to the *magnitude of **A*** <u>times</u> the magnitude of the component of **B** *perpendicular to **A***. OR:

It gives you a vector whose magnitude is equal to the *magnitude of **B*** <u>times</u> the magnitude of the component of **A** *perpendicular to **B***.

The direction of the cross product is perpendicular to the plane defined by **A** and **B**.